Proof that squaring a number consisting of all sixes results in a number with a predictable pattern made up of so many 4, 3, 5 and 6's

Objective

To prove that when a number consisting of a string of n sixes is squared, the resulting number will be made up of n-1 fours followed by a single three followed by n-1 fives followed by a single six.

For example	6^2 =	36	(n=1)
	66^2 =	4356	(n=2)
	666^2 =	443556	(n=3)
	6666^2 =	44435556	(n=4)
	66666^2 =	4444355556	(n=5)

ie $[6\ldots\ldots6]^2 = [4\ldots\ldots4]\ [3]\ [5\ldots\ldots5]\ [6]$

$\leftarrow n \rightarrow$ $\leftarrow n\text{-}1 \rightarrow$ $\leftarrow n\text{-}1 \rightarrow$

Introduction

This proof uses the formula for the sum of a geometric sequence:

$$s = \frac{a \times (1 - r^n)}{1 - r}$$

Where a is the 1st term, r is the multiplying factor and n is the number of terms.

For example (3 + 6 + 12 +24 + 48) = 3 x $(1 - 2^5) / (1 - 2)$ = 3 x (-31) / (-1) = 93

and (6 + 60 + 600 + 6000) = 6 x $(1 - 10^4) / (1 - 10)$ = 6 x (-9999) / (-9) = 6666

Proof

The theory postulates that the result of squaring a number consisting of n sixes is of the form

[4…..$\leftarrow$ n-1 $\rightarrow$…..4] [3] [5..... $\leftarrow$ n-1 $\rightarrow$..…5] [6]

which by constructing as a series of sums and reversing the order of the terms can be expressed as

6
+ 10 x (5….5)
+ 10^n x 3
+ 10^{n+1} x (4….4)

Now, because a number such as 5…5 is a sum of a geometric sequence (5+50+500+5000….) we can use the formula for calculating the sum of a geometric sequence, so we get

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$$6 + 10\times\frac{[5\times(1-10^{n-1})]}{1-10} + 10^{n}\times 3 + 10^{n+1}\times\frac{[4\times(1-10^{n-1})]}{1-10}$$

For simplification, let A = 10^n , which means that 10^{n-1} =A/10 and 10^{n+1} = 10A

$$6 + \frac{50}{9}\times\left(\frac{A}{10} - 1\right) + 3A + \frac{40A}{9}\times\left(\frac{A}{10} - 1\right)$$

$$\frac{1}{9}\ (\ 54 + 5A - 50 + 27A + 4A^2 - 40A\ )$$

$$\frac{1}{9}\ (\ 4A^2 - 8A + 4\ )$$

$$\frac{4}{9}\ (\ A^2 - 2A + 1\ )$$

$$\frac{4}{9}\ (A - 1)^2$$

$$\left[\frac{2}{3}(A - 1)\right]^2$$

$$\left[\frac{6}{9}(A - 1)\right]^2$$

$$\left[6 \times \frac{1 - A}{1 - 10}\right]^2$$

And replacing A with 10^n

$$\left[6 \times \frac{1 - 10^n}{1 - 10}\right]^2$$

And this is the SQUARED sum of a geometric sequence where

6 is the first term
10 is the multiplying factor
n is the number of terms

$$[6 + 60 + 600 + 6000 + \ldots\ldots]^2 \quad (n\ \ terms)$$

Which is $[6666\ldots.]^2$

QED